299. Proposed by C. N. SCHMALL, 89 Columbia Street, New York City.

The sides of a triangle and the area are in arithmetical progression. Find their values, and show there is only *one* solution in rational integers.

Solution by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

To avoid fractions, take the sides and area to be, in order, $2x$, $2x+2y$, $2x+4y$, and $2x+6y$; then $3(x+y)$=the half sum of the sides, from which we have

$$[3(x+y)(x+3y)(x+y)(x-y)]^{\frac{1}{2}}=2x+6y\ldots(1).$$

Throw off the radical and divide each side of equation (1) by $(x+3y)(x+y)^2$ and we have after reduction

$$3(x-y)/(x+3y)=4/(x+y)^2=m^2\ldots(2).$$

The least value of m for positive integral results=1. Therefore $x=2-y$, and the sides and area in order are, $4-2y$, 4, $4+2y$, and $4+4y$.

The least value of y for positive integral results=$\frac{1}{2}$. Therefore, the sides and area are in order 3, 4, 5, and 6.

The triangle is a right triangle; and there are an indefinite number of similar triangles; integral or fractional multiples; but there is but *one* solution.

Also solved by George W. Hartwell, T. I. Wodo, G. B. M. Zerr, M. V. Spunar, A. H. Holmes, J. Scheffer, and J. M. Arnold.

172. Proposed by DR. L. E. DICKSON, The University of Chicago.

Without solving the algebraically solvable quintic, $y^5+py^3+\frac{1}{5}p^2y+r=0$, prove that it is irreducible in the domain of rationality (p, r).

Solution by H. S. VANDIVER, Bala, Pa.

Put the function in the form

$$5y^5+5py^3+p^2y+5r.$$

If the original function is reducible in domain (p, r) this function is also. The assumption that it is reducible in domain (p, r) is equivalent to the assumption that it can be expressed as the product of two factors:

$$x^n+\alpha_1x^{n-1}+\ldots+\alpha_n,$$
$$5x^{5-n}+\beta_1x^{4-n}+\ldots+\beta_{5-n},$$

where the α's and β's are rational functions in p and r. By Theorem VI, page 79, Vol. I, of Weber's *Algebra*, French edition, they may also be considered integral. The form of the factors shows that the function remains